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Three New Aid Mechanisms

Ruth Levine and Steve Radelet

Constrained After College: Student Loans and Early Career Occupational Choices

Jesse Rothstein and Cecilia Rouse

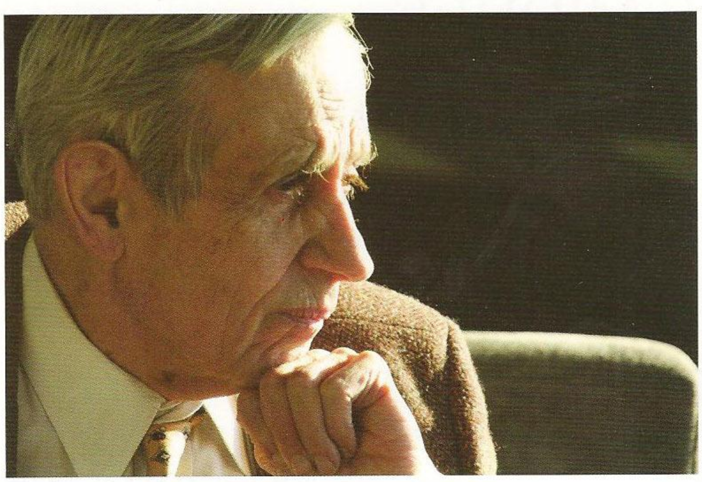
The Causes and Consequences of Industry Self-Policing

Michael W. Toffel and Jodi L. Short

PLUS Great Minds in Economics: John Nash

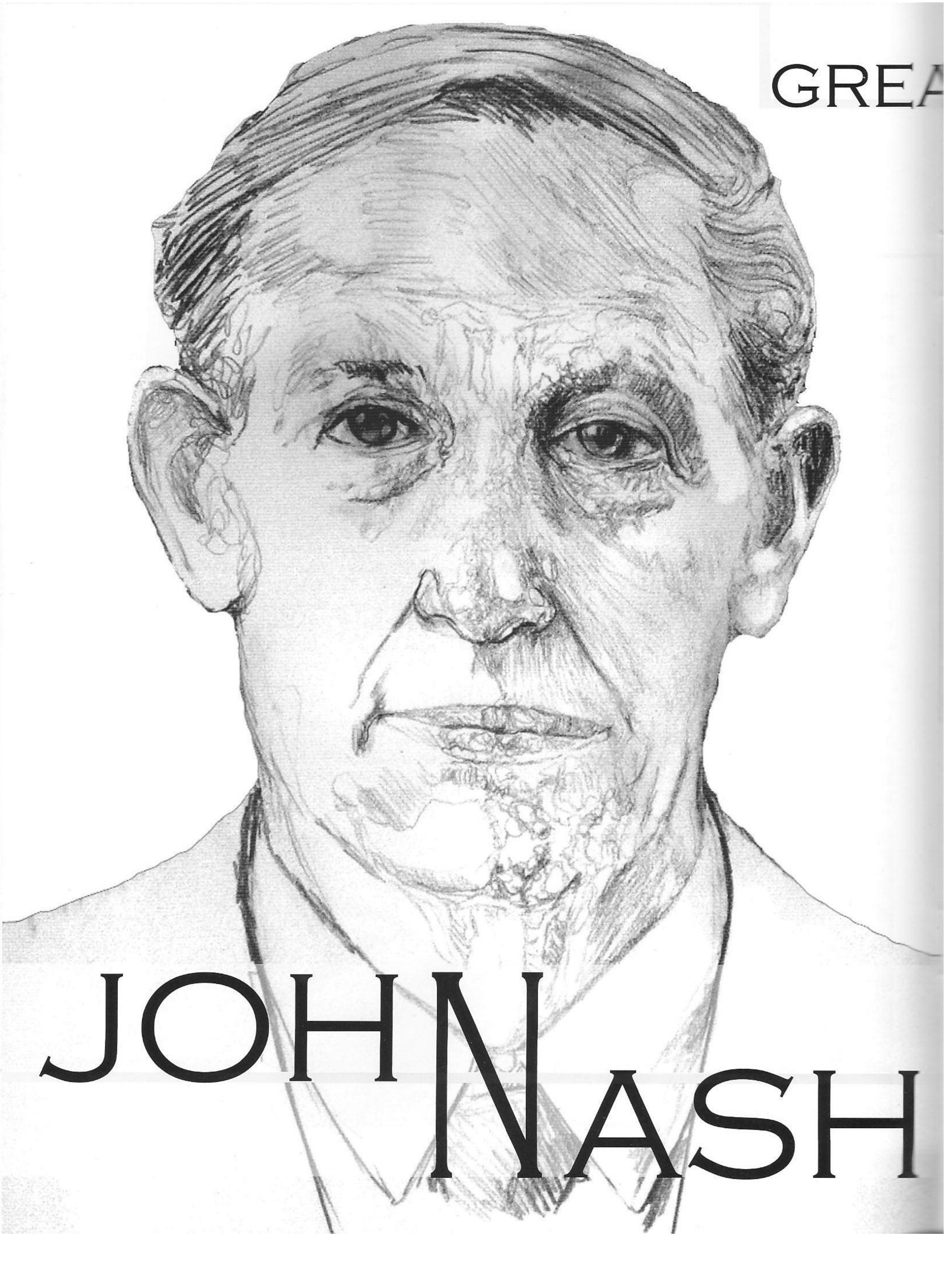
16 Election 2008

The slumping economy has become the primary focus of millions of American voters in the upcoming election. YER offers an economic perspective on the candidates' positions on four critical issues: health care, immigration, tax policy, and the mortgage crisis.



26 Great Minds in Economics

Mathematician and Nobel Laureate John Nash discusses the birth of the Nash equilibrium, his triumph over personal struggles, and the inner workings of "a beautiful mind."



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JOHN NASH

T MINDS IN ECONOMICS

John Forbes Nash, Jr., referred to by Russian mathematician Mikhail Gromov as the "most remarkable mathematician of the second half of the twentieth century," is familiar to many through the best-selling book and Academy-Award winning movie based on his life. A few years after his brilliant thesis, entitled "Non-Cooperative Games," earned him a doctorate at Princeton, Nash began to struggle with paranoia and depression – what he calls "disturbed thinking" and what doctors would later diagnose as schizophrenia. Sylvia Nasar, German economist and author of *A Beautiful Mind*, notes that, for a time, Nash was "frozen in a dreamlike state," living "quietly in Princeton for many years."

Forty years after completing his graduate work, Nash emerged from the fetters of his schizophrenia to win the 1994 Nobel Prize in Economics. His pioneering work in a new branch of economics known as game theory has been used by evolutionary biologists to determine survival strategies, anthropologists to investigate cultural differences in human behavior, philosophers considering the tension between ethical action and self-interest, law professors anticipating legal settlements, and, of course, economists studying all types of strategic interactions. Nash's insights are part of the tool kit used by intelligence analysts at the CIA, political campaign managers, and high priced consultants to Fortune 500 companies. In short, his work has united a score of academic disciplines in an effort to understand and predict human actions.

Still very sharp as he nears his 80th birthday, Nash, senior research mathematician at Princeton, sat down to share his some of his thoughts, memories, and theories with YER.

The Beginnings of Game Theory

In 1944, mathematician John von Neumann and economist Oskar Morgenstern collaborated on an ambitious project. They wanted to mathematically define outcomes of human interaction under the scope of a general theory. To do this, they first resuscitated the concept of expected utility, which had been presented by the physicist Daniel Bernoulli in 1738 as a solution to the St. Petersburg paradox.

In the St. Petersburg game, a player tosses a coin and receives $\$2n$ where n is the number of consecutive heads. The game, which ends with the first toss of tails, has infinite expected value since it is possible, in principle, to toss heads forever. The conventional wisdom of the time was that people should pay the expected value of a gamble to play the game. Bernoulli, however, reasoned that people have a utility function that determines their happiness and that this function increases at a decreasing rate – a concept known as diminishing marginal utility. Bernoulli contended that people would not pay the expected value of a gamble but rather their expected utility (or expected happiness), which would be less than the infinite expected value. "Only a fool would be willing to pay more than 100 pounds," Bernoulli claimed.

For the next two centuries, Bernoulli's theory lay dormant. It was not until von Neumann and Morgenstern touted Bernoulli's idea that it became part of the bedrock of economics. Von Neumann and Morgenstern were able to demonstrate that Bernoulli's diminishing marginal utility arose as a direct consequence of three plausible mathematical assumptions. This supported the hunches of von Neumann and Morgenstern who suspected that there were mathematical principles that governed human actions and enabled the duo to lay the foundations for game theory – a visionary framework to mathematically predict human behavior. A game, as defined in game theory, consists of an interaction between two or more players where each player has a well-defined set of possible strategies and a well-defined set of payoffs given those strategies. The payoffs for each individual player tend to be dependent on the actions of all players in the game.

Given this construction of a game, von Neumann and Morgenstern set out to find the equilibrium strategies in every possible game. In other words, given a game, what strategies would each player play, assuming that each player acts in accordance with expected utility theory? If players do behave according to expected utility theory, then their goal would simply be to maximize their payoffs (and thus maximize their utility). The problem was that solving the games proved in some cases to be much harder than constructing them. To aid their efforts, von Neumann and Morgenstern defined two types of strategies: a strictly dominant strategy for Player 1 was defined to be a strategy that yielded a higher payoff for Player 1 than any of Player 1's other strategies for every possible outcome, regardless of what other players did. A strictly dominated strategy for Player 1 was defined to be a strategy that yielded a lower payoff for Player 1 than any set of Player 1's other strategies in every possible outcome.

Finding strictly dominant strategies enabled von Neumann and Mortgenstern to identify equilibria in games where at least one player had a single best option regardless of what the other players did. However, a large class of games existed in which no player had a strictly dominant strategy. By eliminating strictly dominated strategies, von Neumann and Mortgenstern could identify equilibria in games where a single optimal strategy emerged after sub-optimal strategies were discarded. However, there was still a very large class of games for which no strictly dominated strategies existed. Also, there were other games for which eliminating strictly dominated strategies would result in a game with many strategies which were neither strictly dominant nor strictly dominated. In such games, no equilibrium could be determined.

Von Neumann and Morgenstern summarized their findings in 1947 in their classic *Theory of Games and Economic Behavior*. With their research, the pair could find equilibrium outcomes for two-person zero-sum games. A zero-sum game is a game in which a gain by one player corresponds to an equal loss by the other player. Most games, however, are not zero-sum. Many games also involve more than two players. For the next four years, the foundations of game theory remained incomplete. Was it possible that so many games simply did not have an equilibrium outcome?

The answer to this question, as presented in 1951 in the proceedings of the National Academy of the Sciences, was heralded as the dawn of a new era in economic analysis. A graduate student in mathematics at Princeton had answered this question with a resounding "No!" Any finite, non-co-operative game with any number of players with well-defined payoffs and well-defined strategies had to have at least one equilibrium. For this result to hold, mixed strategies in which players alternate among several preferred strategies to improve their overall performance had to be considered. The essence of this equilibrium was astonishingly simple. The equilibrium signified an outcome in which no player could become better off by deviating and changing his strategy, provided that the other players did not change their strategies. This presented a decentralized mechanism for determining outcomes based on strategic interaction that Nasar describes as a "far more sophisticated version of Adam Smith's great metaphor of the Invisible Hand." The foundations of game theory were now complete. The Princeton graduate student was John Forbes Nash, Jr. and in publishing his paper, which was less than three-quarters of a page long, he had revolutionized the field.

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The Nash Equilibrium

Many of history's great discoveries have been made by visionary twenty-somethings. Isaac Newton was 24 when he reached his conclusions on gravity. Albert Einstein formulated his Theory of Special Relativity at 26. Nash was just 21 when he wrote a brief paper about equilibrium points in *n*-person games. "In mathematics, it is notorious that people tend to do good things typically when they are younger," Nash acknowledged. "They are suspected of being phased out when they are past forty. There's actually a famous prize called the Fields medal in Mathematics, which can only be given to people who are under forty. And one person at Princeton, Andrew Wiles, failed to get it when he proved Fermat's last theorem, which was an extremely famous 300-year problem. He submitted the proof when he had just passed forty."

With regard to his doctoral work, Nash said, "The most challenging part was having an original idea - something which can be developed. I had a breakthrough and a realization with the fixed point theorem – a generalization of the two-person solution from von Neumann, the mini-max solution. It turns out you can derive the two-person theory from it."

Von Neumann, however, was initially unimpressed with Nash's findings. Nash reminisced, "I had been studying mathematics and topology at Princeton and the fixed point theorem. It's a very basic theorem in topology, so when I went to talk with von Neumann…to present the idea to him, he said, 'Did you do this with the fixed point theorem?' and I said 'Yes.'" According to Nasar, von Neumann remarked, "That's trivial, you know. That's just a fixed point theorem."

Despite von Neumann's initial indifference, the Nash equilibrium, now pervasive in the social sciences, has proved extremely useful in predicting the outcomes of strategic interaction. Originally used by a small community of Cold-War analysts to examine the possibility of nuclear war, game theory and the Nash equilibrium have become increasingly embedded in the modern discourse on human behavior across many disciplines, as noted earlier. Some literature professors, for example, have even examined *Romeo and Juliet* and *Hamlet* as a game of imperfect information.

Nash did sense that his concept of equilibrium could be useful. "I knew some applications," he remarked. "I knew of connections which certainly weren't immediately appreciated – connections to cooperative games, collective bargaining, and two-person games." He could never have suspected, however, how widely applied the ideas proposed in his "Non-Cooperative Games" would become.

The Nobel Prize

When asked about winning the Nobel Prize, Nash said, "For some people it's more of a surprise. You think, 'Of course I could do it. I could be one of the lucky winners. But I don't expect it right now.'"

In Nash's case, he had reason to think he was on the selection committee's short-list. "I had heard rumors and I knew before the rumors it would be theoretically possible, because someone had gotten the prize for general equilibrium – Gerard Debreu," said Nash. "There are many examples of people who are not traditionally economists who got the economics prize – Vernon Smith, and a professor in Psychology at Princeton, Daniel Kahneman. So I've known it was possible in principle. And I had published in *Econometrica* as well as in Mathematics journals. Someone here a professor emeritus…told my wife it was to occur before I was told. And then the call (from Stockholm) came in at 6:00 a.m."

A Beautiful Mind

While the Nobel Prize symbolized the academic community's recognition of Nash's work, the popularity of the book and movie *A Beautiful Mind* helped him win the hearts of the general public. "The book and the movie would not have existed if it was not for the Nobel Prize," he said, noting that the prize really changed his life. He recalls that, in the aftermath of winning the Nobel, he finally emerged from years of what he called a "troubled mind." "I was on a tour. I had become a rational thinker," he recounted, referring to the change in his mindset. "Before that, I was a pretty normally behaved person but with disturbed thinking. Then I had even more disturbed thinking and more extravagant behavior." At this point, Nash noted, that Nasar, an economic journalist for the *New York Times*, got the idea for a writing a book about his life. "I didn't cooperate on that book," Nash said. "…I simply wasn't ready for it."

Nash has said that if he were to write a book on his life, "it would have been entirely different" than *A Beautiful Mind*. Nash admitted, though, that he's never "thought seriously about writing a book" on his life. "I've just sort of meditated about what I could do," said Nash. "I could write a book about what I was thinking about – my impressions could play a large part of it. That part is not seen from the outside. There's a mental and psychological area. That would be good to see some of that about a person, especially a person with an interesting mental history. But if I were to write that, it wouldn't be easy."

Meeting with Einstein

While it wouldn't be easy, Nash would certainly have a lot to write about. When he was a doctoral student, Princeton was peopled with some of the most imaginative minds of the twentieth century. Von Neumann was developing many useful mathematical techniques for quantum physics, in addition to his work on game theory. The famed mathematician and logician Kurt Godel was a faculty member, and, of course, Albert Einstein had an office at the Institute for Advanced Study.

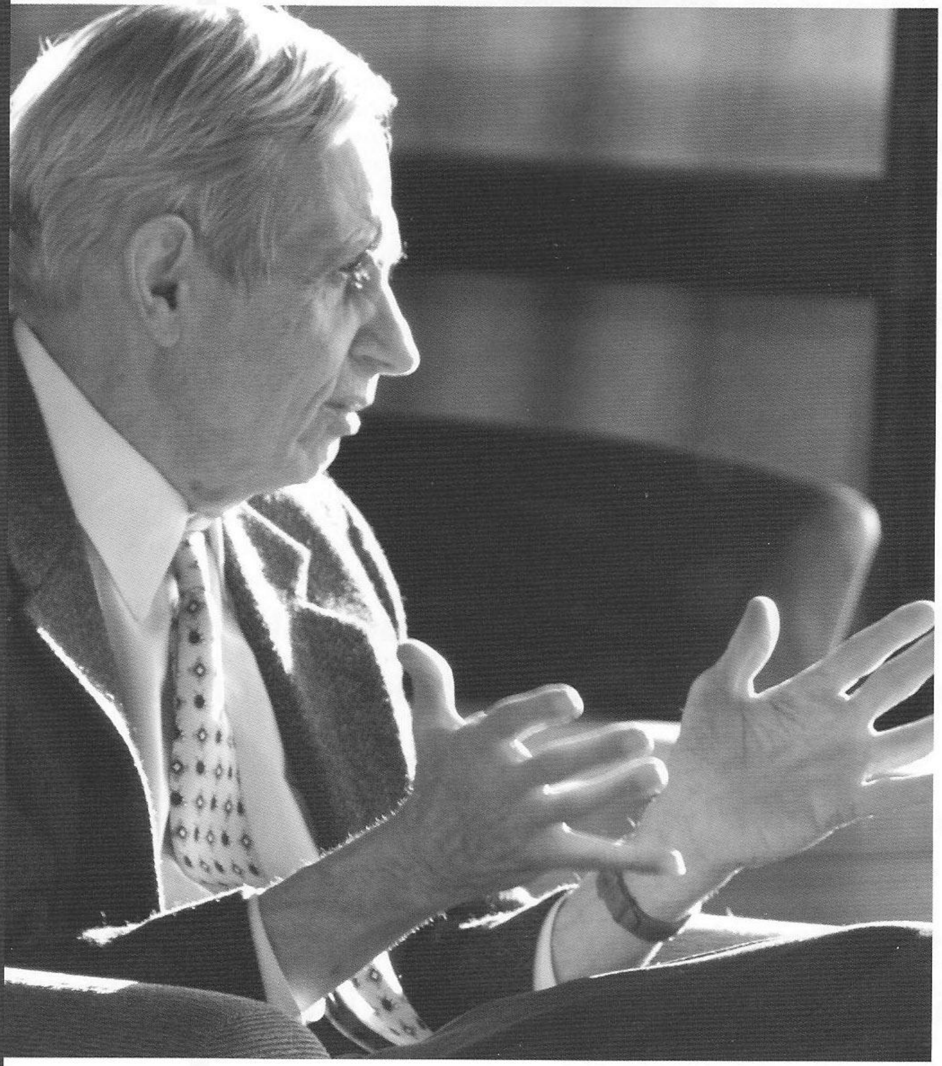
In addition to his encounter with von Neumann, Nash also had the opportunity to meet the frizzy-haired scientist. "Einstein was around Princeton," Nash remembered. "At one time, during my last year in Princeton, I was sitting on the same street where his house was – Mercer Street. His house was actually closer to town. When I was walking to the university, he'd be walking to his office. So, I would pass him by on the sidewalk."

Nash decided to talk to Einstein about some ideas he had for advancing relativity theory. "I went and sought out an interview with him," Nash said. "But we couldn't achieve too much in a short time. I wasn't a physicist or an astronomer, but I had an idea. I thought that this could have relations to the expansion of the universe, to electromagnetism and gravitation, to all sorts of things. We don't really know everything about the universe. Every now and then someone comes up with a new idea – that the universe bounces and contracts or bounces and expands again. The Big Bang arises after a previous contraction and the universe continues that cycle. Nobody really knows, because even if there was a Big Bang, we are really far from it."

Nash has always been a creative thinker. His thoughts have extended from the foundations of mathematics and frontiers of physics to the patterns underlying human social interactions and the inner workings of the human mind. He has weathered many mental storms, emerging with a remarkably clear set of ideas.

Nash's idea that he discussed with Einstein was an alternate explanation for redshift, which occurs when visible light shifts to the red (lower energy) portion of the electromagnetic spectrum. There are three recognized mechanisms for redshift. The Doppler redshift occurs when light moves away from an observer, altering the perceived frequencies of sound waves. The Hubble redshift is due to the expansion of the universe, as galaxies and star clusters move further away from each other. The gravitational redshift occurs when someone receiving light waves is at a higher gravitational potential energy than the source from which the light is emitted.

"I thought the red-shift might be due to a sort of friction that is affecting light," Nash said. "It takes a few minutes for light to actually reach us from the sun. The speed of light is finite. Now there are a few people who have written papers about this idea. The question is whether we are in an expanding universe. The idea that the universe might not be expanding is something which would make the photons lose energy." Nash also noted, however, that "the Doppler effect seems to explain the redshift very well."



Ultimately, Nash realized that there might be a relationship between the Doppler effect and his notion of a "friction…affecting light." In relating these ideas, Nash remarked, "I found an equation that had sort of a nice appearance, but it was fourth order. It has the effect that Einstein had when he introduced the cosmological constant." The cosmological constant was used by Einstein in his field equations to achieve a static universe. When he found that the universe was expanding, Einstein called the cosmological constant his "greatest blunder." However, with the discovery of dark energy and the observation that the expansion of the universe is accelerating, there has been revived interest in the cosmological constant. Interestingly, Nash's equations had the solutions of Einstein's vacuum equations with any cosmological constant. "That's one coincidence about it," Nash said with a spark of curiosity.

Ideal Money

In recent years, Nash has traveled, giving lectures about his ideas for "ideal money" – a currency that would not be affected by changes in inflation. His work is motivated by his belief that inflation is very inefficient, and although it has the propensity to become chronic in certain situations, that there may be a feasible solution that can eliminate it altogether. "I gave a seminar at Yale about my ideal money topic," Nash said. "James Tobin was…in the audience. He came up with a critical point, and asked 'What about the Great Depression?' See, I had been talking about ideal money, which would not be devalued with changes in inflation. The British devalued in 1931 and the Americans in 1933…and then Keynes developed his general theory inspired by this history, and ultimately it became chronic, this process of inflation and Washington gave up any pretense of a relation to gold in 1971."

Despite his general enthusiasm for abstract ideas, Nash argues that ideal money should be a practical concept and will certainly have practical consequences. Nash's concept of ideal money is grounded on the idea that "money should have the function of a standard of measurement and thus that it should be comparable to the watt or the hour or a degree of temperature." Ideal money, according to Nash, could potentially standardize commerce and quality of life and significantly improve the efficiency of the global economy.

Proving the Unprovable

Nash has also thought about mathematical logic and developing a generalized system of logic in which Kurt Godel's theorem would be provable. A fundamental insight of Godel's theorem, as noted by Douglas Hofstadter in *Godel, Escher, Bach* is that "provability is a weaker notion than truth." "The Godel proposition is something that ordinarily is unprovable, but it's because of some limitations of systems," noted Nash. He drew his inspiration from the work of his aforementioned Princeton colleague Andrew Wiles. "I got stimulated by the ideas for the proof of Fermat's last theorem," he said. "Now I had thought that maybe Fermat's last theorem would not be provable. But of course, then one would not know that it's not provable because proving that it's not provable is not disprovable."

Portrait of the Theorist as a Young Man

According to Nasar, in his youth, Nash was eccentric and enjoyed his time alone. This is not atypical. In *A Beautiful Mind*, Nasar notes that "many great scientists and philosophers, among them Rene Descartes, Ludwig Wittgenstein…Isaac Newton, and Albert Einstein, have had similarly solitary personalities."

Philosophers have long suspected that a fine line separates bouts of madness that often accompany such eccentricity from bouts of genius. Both can be characterized by a quantum leap in thinking – by making some connection that goes beyond what people generally consider. Nash has experienced each type of lightning strike, reaching conclusions that have revolutionized the social sciences as well as ideas that have led to "more disturbed thinking and more extravagant behavior." With regard to his years of troubled thought, Nash has said, "It's something of a mystery. It's a special area where smart thinking and crazy thinking can be related. If you're going to develop exceptional ideas, it requires a type of thinking that is not simply practical thinking."

Indeed, Nash has always been a creative thinker. His thoughts have extended from the foundations of mathematics and frontiers of physics to the patterns underlying human social interactions and the inner workings of the human mind. He has weathered many mental storms, emerging with a remarkably clear set of ideas. The equilibrium concept, like the laws of motion, relativity, and evolution contain basic concepts, is relatively straightforward. Einstein once remarked, "Nature is the realization of the simplest conceivable mathematical ideas." This guiding principle has helped shape man's view of the physical universe. Before his twenty-second birthday, Nash extended this vision to analyze basic human interactions and human nature.

Nash noted that the quest for doing innovative scientific work is like the challenge of composing good music. First, you must find your taste, and then you need to develop your style. This captures the essential notion that there is something inherent in creativity which can't easily be taught. It must be lived through experience.

Creativity, however, may not be unique to scientific pioneers like Nash. When asked about genius, Nash noted the origins of the word so often associated with him. "The Greeks had a similar concept," he said. According to the Greeks, every person has a guiding spirit (a genie) inside him or her. "Maybe some spirits are more energetic than others," Nash joked. Humor aside, this basic intuition might suggest that anyone can have a burst of inspiration, leading to a stroke of genius. In this way, "writing a thesis in the sciences is similar to an artist composing some music," Nash remarked. While Nash's work certainly has all the qualities of an artistic masterpiece, his life may be the most poignant symphony of all.